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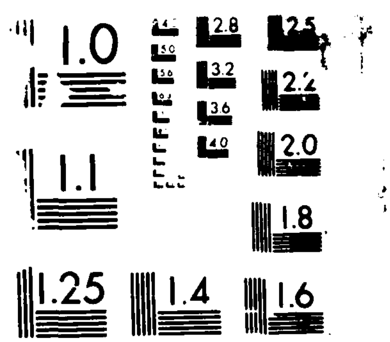

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AN ANALYSIS OF GEOPOTENTIAL EFFECTS ON SATELLITE CONSTELLATIONS

THESIS

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THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
In Partial Fulfillment of the
Requirements for the Degree of
Master of Science in Space Operations

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December 1986

Approved for public release; distribution unlimited

Preface

This study analyzed the relative motion between satellites arranged as a constellation in an attempt to quantify the magnitude of any gravity induced relative motion. It was felt that knowledge of this type of motion could be used by designers of satellite constellations for sizing of propulsion systems for station-keeping and/or for deciding what orbital parameters to use for the satellites within the constellation.

Motion was analyzed between two satellites in circular orbits at altitudes ranging from 300 km to 6000 km and orbit inclinations from 40° to 80°. The J_2 gravitational harmonic was used as the perturbing force, and results from 18 different test cases showed that relative motion does occur as an undamped oscillation in range. However, the magnitude of this oscillation under J_2 perturbations is small enough that the relative motion need not be applied as a constraint on satellite constellation design. The method employed does provide a means of analyzing other perturbing forces, and this could be done to further substantiate the findings of this study.

I wish to extend my thanks to Captain Keith Jenkins from the Systems Concepts Group at AFWAL for identifying this topic as an area which needed investigation. I also

want to thank Lt Col Joe Widhalm for agreeing to be the advisor for this thesis even though I wasn't a "real astro guy."

— Mark J. Buechter

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Abstract

Current plans for systems to be used for ballistic missile defense sometimes call for using satellites that are placed so as to form a constellation that can continuously monitor specified areas of the Earth's surface or direct weapons against attacking missiles or warheads. This study analyzes the relative motion between satellites within such a constellation under gravitational perturbations caused by the Earth's equatorial bulge (oblateness). Relative motion is calculated using a system of equations which describes the variation of relative orbital elements between two satellites. The cases studied simulate the position of two satellites that are located within a constellation containing ten orbits with ten satellites in each orbit. The orbits investigated were all circular with altitudes ranging from 300 km to 1000 km and inclinations ranging from 40 degrees to 80 degrees. Range between the satellites was oscillatory with deviations from the average range of up to 5 km. The results vary only slightly with changes in orbital inclination or altitude. These results show that the relative motion between satellites in a low altitude constellation caused by the Earth's oblateness does not significantly affect the initial geometry of the constellation.

AN ANALYSIS OF GEOPOTENTIAL EFFECTS
ON SATELLITE CONSTELLATIONS

I. Background

President Reagan, in a speech given in March of 1983, proposed that the scientific community of the United States help develop a capability for ballistic missile defense (BMD) to render nuclear weapons "impotent and obsolete" (3:39). This proposal touched off debate throughout the country's scientific community as to how technologically achievable or economically feasible this type of defense system would be.

Both sides in this argument have put forth proposals on how such a system for BMD should be structured. All of these options call for some type of airborne or spaceborne sensor system to detect incoming missiles/warheads, but the various options differ on how much of the actual weaponry should be placed in orbit.

When orbiting weapons are needed they must be placed in an arrangement that allows continuous coverage of those areas of the Earth's surface over which a ballistic missile attack might occur. When several satellites are placed into orbits so that they can provide complementary capabilities for sensor coverage, data relay or any other purpose, the satellites' position relative to one another will need to be specified. Such arrangements of satellites are commonly referred to as constellations and generally consist of several orbits, all of equal geometry and inclination, with their lines of nodes spaced equally around the Earth's equatorial plane. Each orbit contains several satellites evenly spaced throughout. Figure 1 illustrates such a constellation of satellites.

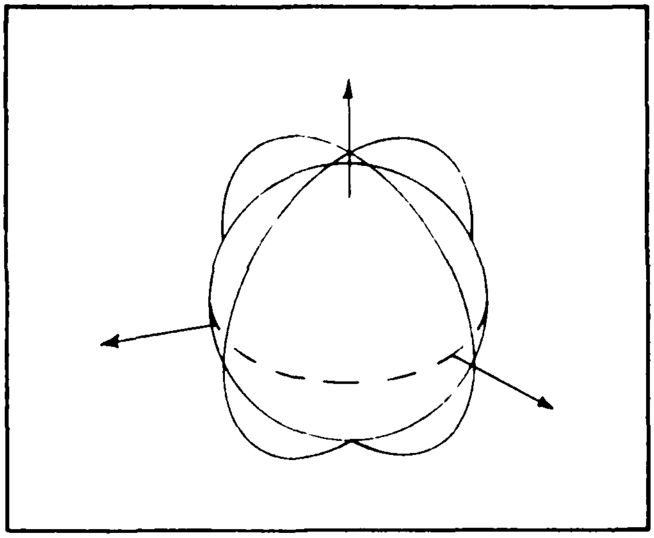

Fig. 1. Illustration of a Satellite Constellation

The satellites may be the actual weapons or in some cases they are mirrors used to redirect a laser beam emanating from a surface based weapon. In either case, the relative positions of the satellites within the constellation will determine the coverage the weapon has over the Earth's surface. Should the satellites' spacing within the constellation become disrupted there is the possibility that a gap may occur in the weapons' coverage. This gap would, of course, be undesirable.

It appears, however, that none of the designers of these constellations have looked at how perturbing forces will affect the constellations. Since all of the current proposals put the satellites at relatively low altitudes, the orbits are considered near-earth and will experience perturbations due to the Earth's non-spherical shape. These perturbations could result in oscillations of the relative positions of the satellites within the constellation that may be inconsequential or perhaps will lead to total disruption of the constellation.

Research Problem

It is necessary to know how the actual gravitational field of the Earth will affect the movement of satellites within a constellation to determine if there are adverse changes in the weapons coverage of the total system. There may be some altitudes, inclinations or combinations that need to be avoided because they tend to accentuate the relative motion between the satellites, or that can be capitalized upon to lessen any relative motion problem. If some constellation geometries are deemed vital for the weapon's coverage but run contrary to the satellites' natural motion, appropriate propulsion systems will need to be included in the satellites' design.

Previous Research

A literature search for previous research concerning satellite motion within a satellite constellation revealed nothing directly addressing this problem. However, this initial screening did reveal work on the motion of one satellite that is very close to another (7; 8). This work was directed at applications such as the proposed free-flying microgravity lab to be used with the U.S. space station, but also appeared applicable to the satellite constellation problem. The foundation of the analysis was rendezvous theory, and since the rendezvous problem also involves solving for the relative motion between two orbiting objects, research into the solution methods for the rendezvous problem seemed an appropriate starting point for the constellation problem. Early studies of the rendezvous problem give approximate solutions and deal only with special conditions, but the more recent work gives improvements on these techniques.

One of the earliest attempts at solving the relative motion problem inherent in the rendezvous of two orbiting satellites was accomplished by Eggleston. In his study, Eggleston looked at how to calculate the minimum change in velocity needed to achieve rendezvous between two satellites, and listed several sets of equations which can be used to solve this problem depending upon the amount of accuracy needed (4:27). However, this investigation was limited to circular orbits about a spherical earth (4:10).

Anthony and Sasaki further developed the solution for the rendezvous problem by allowing the use of orbits with small eccentricities and by including second-order terms in the equations (1:1669). These additions to the solution method allowed the problem to be solved for either large or small relative distances (1:1673).

The research by Anthony and Sasaki, and Eggleston provided only approximate solutions to the relative position problem, but Lancaster (5:1878) showed how to solve the problem exactly. Like the previously discussed works, however, Lancaster's solution was also based on a spherical earth assumption.

All of these methods for solving the relative motion problem were based on assumptions that simplified the solution method. The one common assumption was that the satellite was orbiting a spherical planet, and therefore the gravitational field of that planet was uniform. Since the Earth has many nonuniformities in its mass distribution, the primary one being a bulging at the equator, an object in orbit experiences perturbing forces as it passes over different areas of the Earth's surface. To solve for the true motion of the satellite when these forces are added to the satellite's equations of motion requires the use of a computer for numerical integration. Unfortunately, when the two satellites of interest come close together, and their position vectors become almost equal, much precision in the solution is lost because of the requirement to subtract these nearly equal vectors.

Nacozy and Szebehely developed a technique for finding relative position and velocity using numerical integration that retains the precision of the solution when two nearby satellites are studied (7:449). They used the basic technique outlined by Eggleston, but improved it to allow for non-circular orbits (7:449). Nacozy and Szebehely's study was done in conjunction with a study by McKenzie in which eight different sets of equations were evaluated for computational efficiency (6:10). Efficiency was measured by the number of computations done by the computer during numerical solution of the equations. The equations identified in McKenzie's report as Set 6 apply the Nacozy-Szebehely technique to the classic method of Cowell for perturbations and are applicable to the case where the two satellites are in nearly equal orbits (6:15).

Van der Ha's work, the foundation of which can be traced to all of the previously cited works, developed equations for the motion between two satellites requiring no simplifying assumptions. Van der Ha's technique transforms the satellites' position and velocity vectors into six elements which completely describe the relative motion (8:287). There are no limitations on the eccentricities or inclinations of the orbits, or on the distance between the satellites. Van der Ha's study was aimed at the motion between a space station and a subsatellite launched from the station and included perturbations for a non-spherical earth and for air drag (8:287). Since the non-spherical effects did not produce significant changes from the spherical assumption due to the closeness of the satellites, the test cases studied mostly dealt with changes due to air drag (8:299). Nonetheless, the solution method developed by Van der Ha was also applicable to this author's study.

Research Objective

The objective of this study was to determine if perturbations of the orbits of satellites arranged as a constellation cause relative motion between the satellites. If there is relative motion, the magnitude must be determined and the stability of the motion must be evaluated to see if there is any tendency for the satellites to stay in the same general relative position or to drift even further apart.

Scope of the Research

Because the current proposals for satellite constellations to be used for ballistic missile defense (BMD) call for low altitude orbits (less than one earth radius), irregularities in the Earth's gravitational field will be a major perturbing force. Therefore, this investigation focused on gravitational perturbations and, more specifically, was limited to inclusion of the primary oblateness term (the J_2 harmonic) from the mathematical expansion of the geopotential.

Captain Keith Jenkins from the Air Force Wright Aeronautical Laboratories, who originally posed this research question, noted that a constellation of 10 circular orbits with 10 satellites per orbit at an altitude of 1000 km is being used as a baseline by many BMD designers. This constellation geometry was used as a baseline from which several test cases were drawn. The basic investigation looked at the relative motion between two adjacent satellites in the same orbit, and at the motion between two satellites in different, but adjacent, orbits. Then the altitude and inclination of the orbits were varied to determine the sensitivity of the results to these parameters.

II. Methodology

Discussion of Method Used

The method chosen to conduct this study of relative motion between satellites in a constellation was to numerically integrate a set of equations of motion for one satellite and a set of equations describing the change in the state of the second satellite with respect to the first. This method is described by Van der Ha (8) and major points will be described here. The core of this method is the development of a set of difference elements between the two satellites, and equations for the time rate of change of these difference elements. Difference elements are used to avoid losing precision in the solution caused by the subtraction of two nearly equal numbers (8:293). This methodology allows analysis of relative motion regardless of how close the two satellites approach one another. Therefore, this methodology could be applied to satellite constellations containing large numbers of satellites as has been suggested by the distributed assets, or "swarm," concept for deployment of space-based ballistic missile defenses.

To allow for circular and/or equatorial orbits, the state of the satellites is described using a set of orbital elements different from the classical elements. These elements are listed as Equation (1).

$$\vec{a} = (h, f, g, j, k, L) \tag{1}$$

These elements are written in terms of the classical elements as

$$\begin{aligned}
h &= \sqrt{\mu \ell} \\
f &= e \cos \theta \\
g &= e \sin \theta \\
j &= \tan(i/2) \cos \Omega \\
k &= \tan(i/2) \sin \Omega \\
L &= \theta + \omega + \Omega
\end{aligned} \tag{2}$$

where ℓ is the semi-lattus rectum, e is the eccentricity, i is the orbit's inclination, Ω is the longitude of the ascending node, ω is the argument of perigee, and θ is the true anomaly of the satellite.

Conversely, the classical elements can be written in terms of the elements in Equation (1) as

$$\begin{aligned}
\ell &= h^2/\mu \\
e &= \sqrt{f^2 + g^2} \\
\theta &= \arctan(g/f) \\
i &= 2 \arctan(\sqrt{j^2 + k^2}) \\
\Omega &= \arctan(k/j) \\
\omega &= L - \theta - \Omega
\end{aligned} \tag{3}$$

To avoid loss of precision when solving for relative motion, a set of difference elements is used, and are written as

$$\Delta\vec{a} = (\Delta h, \Delta f, \Delta g, \Delta j, \Delta k, \Delta L) \tag{4}$$

These difference elements are defined as the difference between the orbital elements of the two satellites where one of the satellites is designated as the reference. The relationship between the positions of the two satellites is shown in Figure 2 where the coordinate system $OX_1X_2X_3$ is an inertial frame centered at the Earth's mass center and the X_1 and X_2 axes are in the equatorial plane.

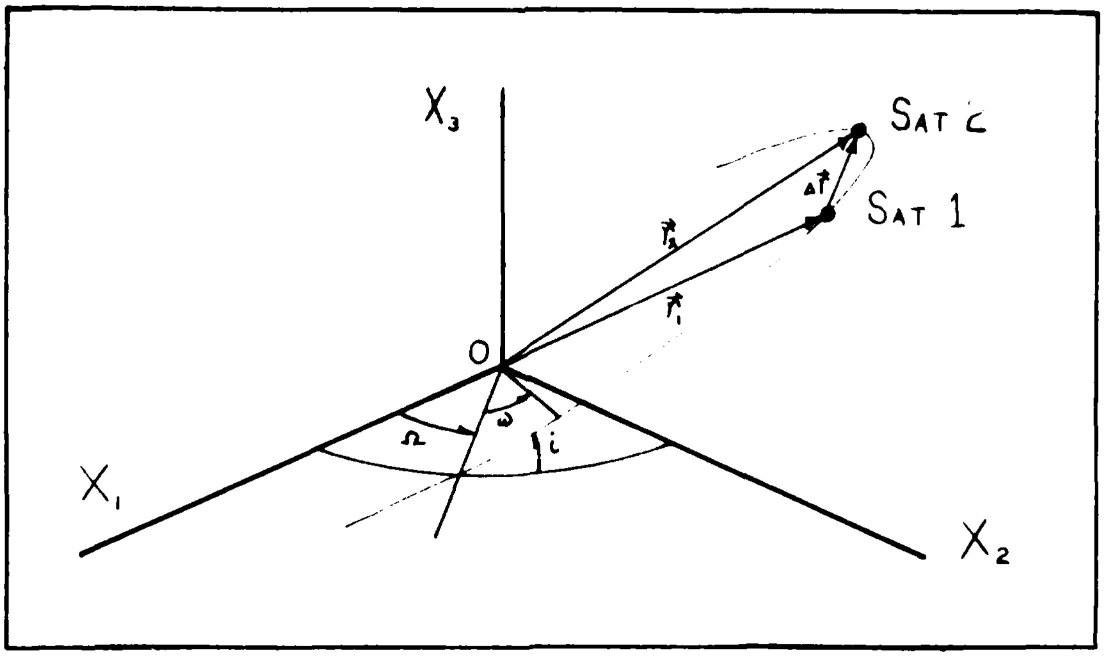

Fig. 2. Relationship Between Position Vectors of Two Satellites

Therefore one can write

$$\vec{r}_2 = \vec{r}_1 + \Delta\vec{r} \tag{5}$$

or

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 \tag{6}$$

This relationship can also be applied to the element sets so that

$$\Delta\vec{a} = \vec{a}_2 - \vec{a}_1 \tag{7}$$

and therefore

$$\begin{aligned} \Delta h &= h_2 - h_1 \\ \Delta f &= f_2 - f_1 \\ \Delta g &= g_2 - g_1 \\ \Delta j &= j_2 - j_1 \\ \Delta k &= k_2 - k_1 \\ \Delta L &= L_2 - L_1 \end{aligned} \tag{8}$$

A set of six differential equations can be written describing the change in the orbital elements with time which include the components of the perturbing force on the satellite. These force components are with respect to a reference frame centered at the satellite with the x-axis along the position vector from the Earth to the satellite, the z-axis along the orbit normal, and the y-axis in the direction which completes the right-handed triad as shown in Figure 3.

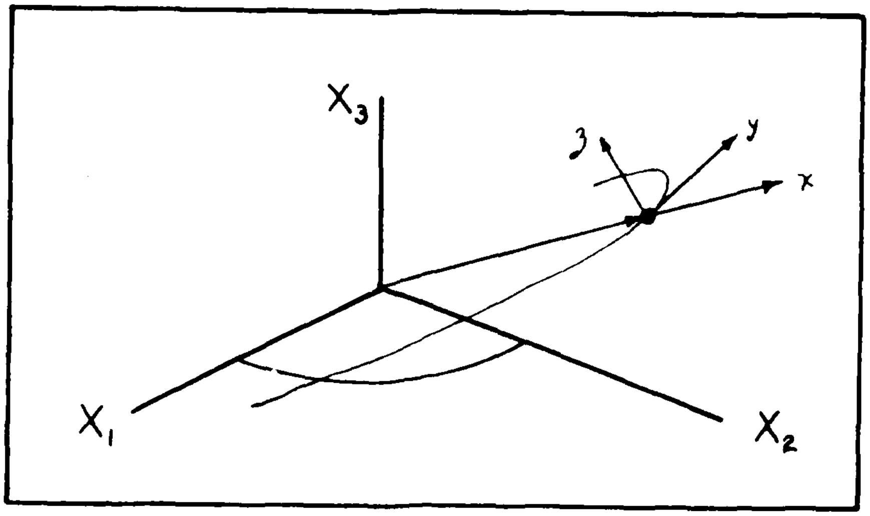

Fig. 3. Satellite Centered Reference Frame

Development of these equations is through the technique of variation of parameters, and the six equations are as follows (8:292)

$$\dot{h} = rF_y$$
$$\dot{f} = -hg/r^2 + 2hF_y/\mu$$
$$\dot{g} = hf/r^2 + hF_x/\mu + grF_y/h$$
$$\dot{j} = rF_z \cos L\,(1+j^2+k^2)/(2h)$$
$$\dot{k} = rF_z \sin L\,(1+j^2+k^2)/(2h)$$
$$\dot{L} = h/r^2 + rF_z(j \sin L - k \cos L)/h \qquad (9)$$

where

$$r = h^2/[\mu(1+f)]$$

and F_x, F_y and F_z are the components of the perturbing force along the corresponding local axes shown in Figure 3 (8:292).

The equations for the variation of the relative elements are found by subtracting the equations for the two satellites. For example

$$\Delta\dot{f} = \dot{f}_2 - \dot{f}_1 = -\Delta(hg/r^2) + 2\Delta(hF_y)/\mu \tag{10}$$

Expansion of this equation without loss of precision is done by using the following procedures where x and y denote arbitrary variables (8:293),

$$\begin{aligned}
\Delta(x^2) &= (2x+\Delta x)\Delta x \\
\Delta(xy) &= (y+\Delta y)\Delta x + x\Delta y \\
\Delta(xyz) &= (z+\Delta z)\Delta(xy) + xy\Delta z \\
\Delta(x/y) &= [\Delta x-(x/y)\Delta y]/(y+\Delta y)
\end{aligned} \tag{11}$$

A partial application of these rules to Equation (10) leads to

$$\Delta\dot{f} = -[(g+\Delta g)\Delta(h/r^2) + h\Delta g/r^2] + 2\Delta(hF_y)/\mu \tag{12}$$

where the first term in Equation (10) has been grouped so that $x = h/r^2$ and $y = g$ for use with Equations (11).

Similar applications of these rules allow one to derive the following full-precision equations for the variation of all of the difference elements (8:293)

$$\Delta\dot{h} = \Delta(rF_y)$$

$$\Delta\dot{f} = -[(g+\Delta g)\Delta(h/r^2) + h\Delta g/r^2] + 2\Delta(hF_y)/\mu$$

$$\Delta\dot{g} = [(f+\Delta f)\Delta(h/r^2) + h\Delta f/r^2] + \Delta(hF_x)/\mu + [(g+\Delta g)\Delta G_y + G_y\Delta g]$$

$$\Delta\dot{j} = \tfrac{1}{2}[(1+J+\Delta J)\Delta G_z + G_z\Delta J]\cos(L+\Delta L) + \tfrac{1}{2}G_z(1+J)\Delta(\cos L)$$

$$\Delta\dot{k} = \tfrac{1}{2}[(1+J+\Delta J)\Delta G_z + G_z\Delta J]\sin(L+\Delta L) + \tfrac{1}{2}G_z(L+J)\Delta(\sin L)$$

$$\Delta\dot{L} = \Delta(h/r^2) + (j\sin L - k\cos L)\Delta G_z + (G_z+\Delta G_z)[\Delta j\sin(L+\Delta L) - \Delta k\cos(L+\Delta L) + j\Delta(\sin L) - k\Delta(\cos L)] \quad (13)$$

where

$$J = j^2 + k^2$$

$$G_j = rF_j/h, \quad j=y,z$$

$$\Delta J = \Delta(j^2) + \Delta(k^2)$$

$$\Delta G_i = [\Delta(rF_i) - G_i\Delta h]/(h+\Delta h), \quad i=y,z$$

$$\Delta r = [\Delta(h^2) - h^2\Delta f/(1+f)]/[\mu(1+f+\Delta f)]$$

$$\Delta(h/r^2) = [\Delta h - h\Delta(r^2)/r^2]/(r+\Delta r)^2$$

It should be noted that the difference force components ΔF_x, ΔF_y and ΔF_z should not be interpreted as the vector difference of the two perturbing forces, $\vec{F}_2 - \vec{F}_1$, written in the local reference frame of Satellite 1. Rather, each difference force component is simply the scalar difference of the corresponding components of each vector written in its own local frame (8:292). For example, if the local unit vectors for Satellite 1 are $\hat{x},\hat{y},\hat{z}$ and the local unit vectors for Satellite 2 are $\hat{i},\hat{j},\hat{k}$, the local perturbing forces would be written as

$$\vec{F}_1 = X\,\hat{x} + Y\,\hat{y} + Z\,\hat{z}$$

$$\vec{F}_2 = I\,\hat{i} + J\,\hat{j} + K\,\hat{k} \qquad (14)$$

The difference force components are then

$$F_x = I - X$$

$$F_y = J - Y$$

$$F_z = K - Z \qquad (15)$$

The perturbing forces themselves are derived from an equation for the potential energy of the satellite. This potential function follows from Poisson's equation and can be written in spherical coordinates as

$$V(r,\theta,\phi) = \frac{GM}{r} \sum_{n=0}^{\infty} \sum_{m=0}^{n} (R_e/r)^n\, P_n^m(\cos\theta)$$

$$\times\ [S_{nm} \sin(m\phi) + C_{nm} \cos(m\phi)] \qquad (16)$$

where r is the length of the position vector from the center of mass of the Earth to the satellite, θ is the co-latitude and ϕ is the longitude as shown in Figure 4 (9:57).

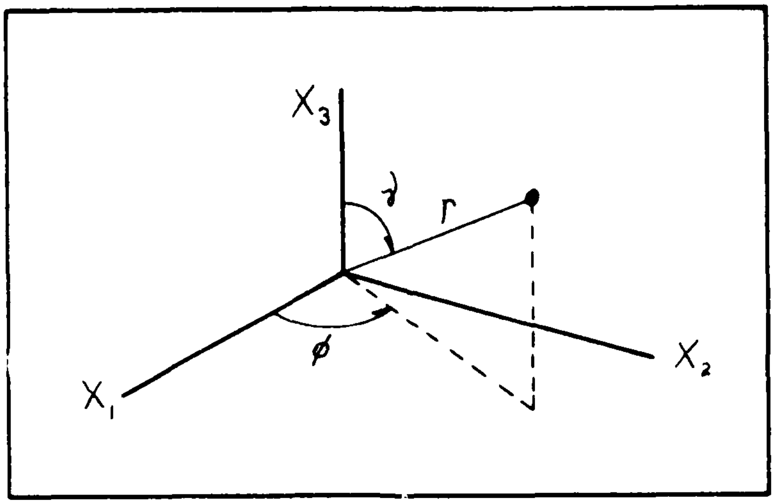

Fig. 4. Definition of Co-latitude and Longitude

Other constants appearing in the geopotential function are the universal gravitational constant, G, the mass of the Earth, M, and the Earth's mean equitorial radius, R_e.

The perturbing force is then calculated as

$$\vec{F} = -\vec{\nabla} V$$

The potential function can be written in terms of the inertial cartesian coordinates by observing from Figure 4 that $\cos\gamma = X_3/r$. Thus Equation (16) can be written

$$V = -3\mu R_e^{\,2} J_2[1/3 - (X_3/r)^2] \,/\, (2r^3) \tag{17}$$

where μ = GM.

Solving for force yields

$$\vec{F} = -\vec{\nabla}V = -\varepsilon[1-5(X_3/r)^2]\vec{r}/r^5 - 2\varepsilon X_3\hat{U}_3/r^5 \quad (18)$$

where $\varepsilon = 3\mu J_2 R_e^2/2$.

However, the quantities F_x, F_y and F_z needed in Equations (5) are components of the force vector written in the local frame, so the force vector must be transformed into the local reference frame. This transformation can be written as

$$\begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} = [A] \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} \quad (19)$$

where U_i are the components of a vector in the inertial frame and u_i are the local frame components. The components of the transformation matrix are (8:306)

$$\alpha_{11} = k\alpha_{13} + \cos L$$

$$\alpha_{12} = -j\alpha_{13} + \sin L$$

$$\alpha_{13} = 2(j \sin L - k \cos L)/(1+J)$$

$$\alpha_{21} = k\alpha_{23} - \sin L$$

$$\alpha_{22} = -j\alpha_{23} + \cos L$$

$$\alpha_{23} = 2(j \cos L + k \sin L)/(1+J)$$

$$\alpha_{31} = 2k(1+J)$$

$$\alpha_{32} = -2j/(1+J)$$

$$\alpha_{33} = (1-J)/(1+J) \qquad (20)$$

Performing this transformation yields (8:298)

$$F_x = -\varepsilon(1-3\alpha_{13}^{2})/r^4$$

$$F_y = -2\varepsilon\alpha_{13}\alpha_{23}/r^4$$

$$F_z = -2\varepsilon\alpha_{13}\alpha_{33}/r^4 \qquad (21)$$

The difference components are found using the relationship

$$\Delta F_i = (F_2)_i - (F_1)_i, \quad i = x,y,z \qquad (22)$$

and the relationships of Equations (11).

$$\Delta F_x = \varepsilon[3\Delta(\alpha_{13}^{2}) + (1-3\alpha_{13}^{2})\Delta(r^4)/r^4]/(r+\Delta r)^4$$

$$\Delta F_y = 2\varepsilon[\alpha_{13}\alpha_{23}\Delta(r^4)/r^4 - \Delta(\alpha_{13}\alpha_{23})]/(r+\Delta r)^4$$

$$\Delta F_z = 2\varepsilon[\alpha_{13}\alpha_{33}\Delta(r^4)/r^4 - \Delta(\alpha_{13}\alpha_{33})]/(r+\Delta r)^4 \qquad (23)$$

Implementation of Equations

To solve the equations for the variation of the orbital elements and the difference elements a computer program was written using Fortran 77 and a numerical integration package. The numerical integrator employed was a fourth order predictor-corrector called HAMING which was provided by Dr. William Wiesel of the Air Force Institute of Technology, Wright-Patterson Air Force Base, Ohio. This program provided a time history of the orbital elements of Satellite 1 and of the difference elements, and was verified using two test cases whose results could be predetermined. Both verification tests involved unperturbed motion.

The first test case used two satellites in the same orbit but separated by 36° of true anomaly. The orbital parameters in terms of classifical elements were

$$l = 6778 \text{ km} \qquad \omega = 0°$$
$$e = 0 \qquad \Omega = 0°$$
$$i = 45° \qquad \theta_1 = 20°$$
$$\theta_2 = 56°$$

where subscripts denote parameters for the first and second satellites. For unperturbed motion, the separation between the satellites should remain constant because the satellites are in circular orbits. This was noted by observing the distance between the satellites at each time step during the integration. The initial separation distance was calculated using the equation

$$R = 2\ell \sin[(\theta_2-\theta_1)/2] = 4189.034 \text{ km} \qquad (24)$$

where R is the range between the two satellites.

When this first test case was run on the computer program, the distance between the satellites did remain constant at the predicted value of 4189.034 km.

The second verification test case consisted of having two satellites in circular orbits of equal altitude but at different inclinations. The two orbits intersected along their lines of nodes and the test run was begun with both satellites together at the ascending node. The initial conditions were

$$\ell = 6778 \text{ km} \qquad \omega = 0°$$

$$e = 0 \qquad \Omega = 0°$$

$$i_1 = 30° \qquad \theta = 0°$$

$$i_2 = 60°$$

For unperturbed motion the range between the two satellites will oscillate from the initial condition of zero to a maximum value after one-fourth of a period and back to zero after one-half period. This sequence will repeat every half period. The relationship between the satellites after one quarter period is illustrated in Figure 5.

The range at this time is

$$R = 2\ell \sin[(i_2 - i_1)/2] = 3508.551 \text{ km} \tag{25}$$

The time history of this test case is shown in Figure 6.

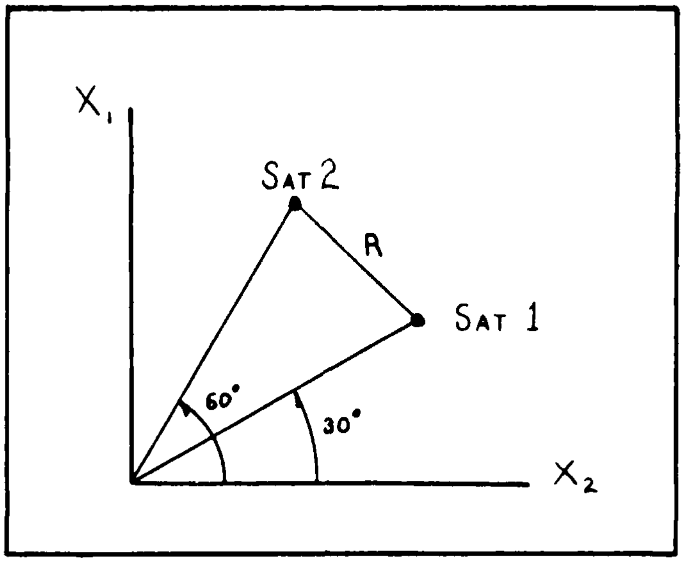

Fig. 5. Range Relationship for Second Verification Test

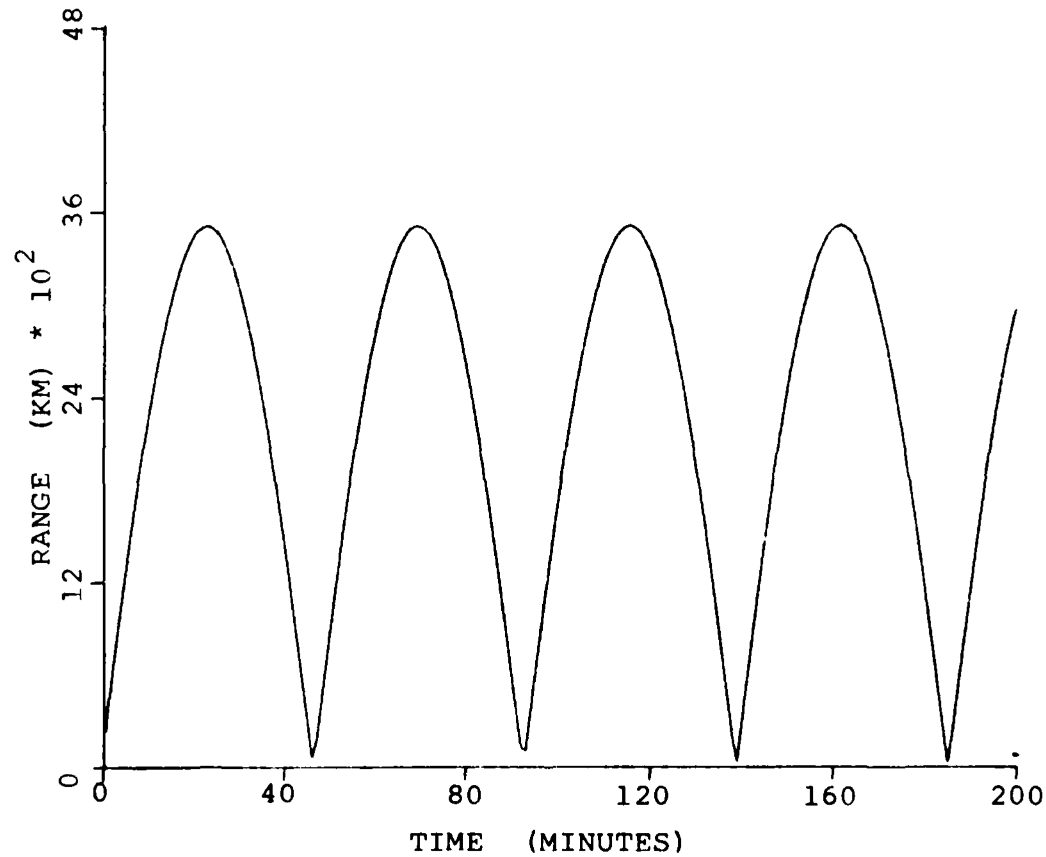

Fig. 6. Output of Second Verification Test

Since there was no real data against which the computer program's output could be compared for perturbed relative motion, the author attempted to verify Van der Ha's force equations by deriving the perturbing force components expressed in the satellite's local frame. However, the force components as expressed by Equations (21) could not be obtained, so intermediate solutions for F_x, F_y, and F_z were implemented in the computer code.

Several cases were run using this new coding for the perturbed force components and the original version of the computer program which used the expressions shown in Equations (21). Identical results were obtained from both versions of the program, so Equations (21) were considered to be accurate and coded properly.

Since the results of the verification runs were as anticipated and the code appeared to contain no syntax errors, the model was considered to be verified and ready for use.

III. Discussion of Results

With the equations programmed and the computer routine verified for proper execution, a series of test cases was developed to examine the relative motion problem and its sensitivity to changes in orbit altitude and inclination. As mentioned in the opening chapter, two basic scenarios were used to evaluate relative motion within a constellation: two satellites in the same orbit but separated by some amount of true anomally, and two satellites at identical positions within two different orbits. In the second scenario the orbits were of identical geometry but had different ascending nodes. While the second scenario geometry could result in satellites being in orbits that would result in a collision (polar orbits), this situation was not investigated. The baseline orbit investigated was circular with a radius of 7378 km and inclined 60 degrees to the equatorial plane.

First Scenario

For the first scenario the two satellites were placed 36 degrees apart within the orbit and the relative motion calculated for ten orbits. The difference in true anomally of 36 degrees between the satellites corresponds to the spacing between te.. satellites equally spaced within one orbit. The results of this test case using the

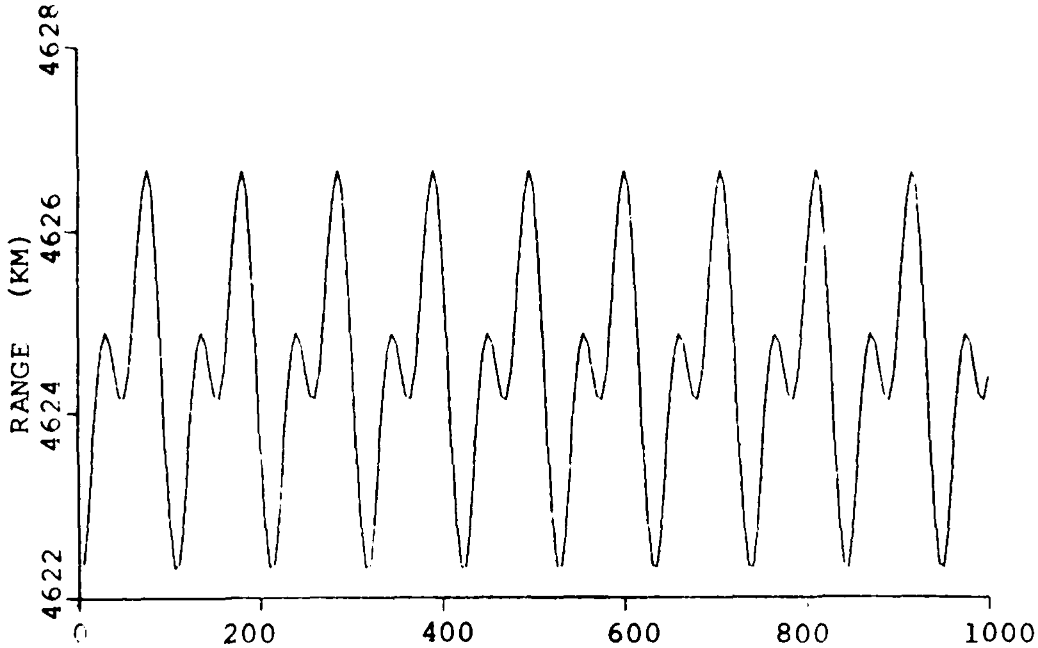

Fig. 7. Baseline Case, First Scenario

baseline orbit are shown in Figure 7, where range is simply the magnitude of the relative position vector between the two satellites.

Using Equation (24) the unperturbed range was calculated to be

$$R = 2\,\ell\,\sin[(\theta_2-\theta_1)/2] = 4560 \text{ km} \tag{26}$$

However, as shown in Figure 7 the average value around which the range oscillates remains constant at the value of 4624.5 km. The difference between these two values can be explained in part by the way in which the initial conditions for the computer program were calculated and partly by the fact that the perturbed orbit will be slightly non-circular.

The initial conditions used as input to the main computer program were established by first defining the initial orbit and the satellite's position within it using the classical orbital elements $(a, e, i, \Omega, \omega, \nu)$. These orbital elements, transformed into the element set and defined in Equation (1) and the equations for the variation of the elements, Equations (13), were then integrated for one-tenth of a period. The values for the orbital elements at this time were used as the initial conditions for the first satellite. Integration for another one-tenth of a period yielded another set of orbital elements to use as initial conditions for the second satellite. This was done to ensure that the initial orbital elements for both satellites were in the same perturbed orbit. A vector of these orbital elements, $(\vec{a}_0)_1$ and $(\vec{a}_0)_2$, was then used as the initial state vector for the main computer program.

However, it was not possible to obtain $\vec{a}_{0_1}$ and $\vec{a}_{0_2}$ exactly one-tenth of a period apart because of the integration step size. For example, the total period for an unperturbed circular satellite is found using the equation

$$TP = (2\pi / \sqrt{\mu})\ r^{1.5} \tag{27}$$

where r is the radius of a circular orbit. For an orbit with a radius of 7378 km the period will be 6307 seconds.

If one-tenth of this time is rounded off to 630 seconds, there is a round-off error of .7 seconds which can be transformed into an angular error by multiplying by the angular velocity of the satellite. Angular velocity is calculated as follows:

$$\omega_{SAT} = 360°/6307 \text{ sec} = .05708 \text{ deg/sec} \tag{28}$$

Multiplying ω_{SAT} by the round-off time yields the angular error.

$$\Delta\theta = (\Delta t)(\omega_{SAT})$$

$$= (.7 \text{ sec})(.05708 \text{ deg/sec}) = .03996 \text{ deg} \tag{29}$$

The relationship used in Equation (24) then yields the range error.

$$R = 2\ r\ \sin(\Delta\theta/2)$$

$$= 2\ (7378 \text{ km})\ \sin(.10998) = 5.15 \text{ km} \tag{30}$$

While this particular value is small, it does partially account for the difference between the unperturbed range and the perturbed result.

A further explanation of this discrepancy lies in the evolution of the orbit's eccentricity over time. The time variation of eccentricity under any arbitrary perturbing force is given by the equation (2:401)

$$\frac{de}{dt} = \frac{\sqrt{1-e^2}\,\sin\theta}{na} F_x + \frac{\sqrt{1-e^2}}{na^2 e}\left[\frac{a^2(1-e^2)}{r} - r\right] F_y \tag{31}$$

where n is the mean motion, $\sqrt{\mu/a^3}$, and a is the length of the semi-major axis of the orbit. While this equation has a singularity when eccentricity is zero and therefore cannot be implemented to use with circular orbits, it does show that if there are any force components in the local x-y plane then the eccentricity of the orbit will change over time. For any non-circular orbit one would then expect the relative position to change with time, even for unperturbed motion. Values of eccentricity calculated during the baseline case run were on the order of 10^{-3} to 10^{-4}.

To determine how much of the range oscillation shown in Figure 7 is attributed to the change in the orbit's eccentricity, a case was run using a typical value of eccentricity from the baseline run. The initial conditions for this non-circular case were the same as the baseline (r = 7378 km, i = 60, $\theta_2 - \theta_1 = 36°$) except for the eccentricity which was set at .001. The results are shown in Figure 8.

Comparison of Figures 7 and 8 shows that the outcomes have the same trend for both the initially circular, perturbed case and the slightly eccentric, unperturbed case. From this comparison one can conclude that the

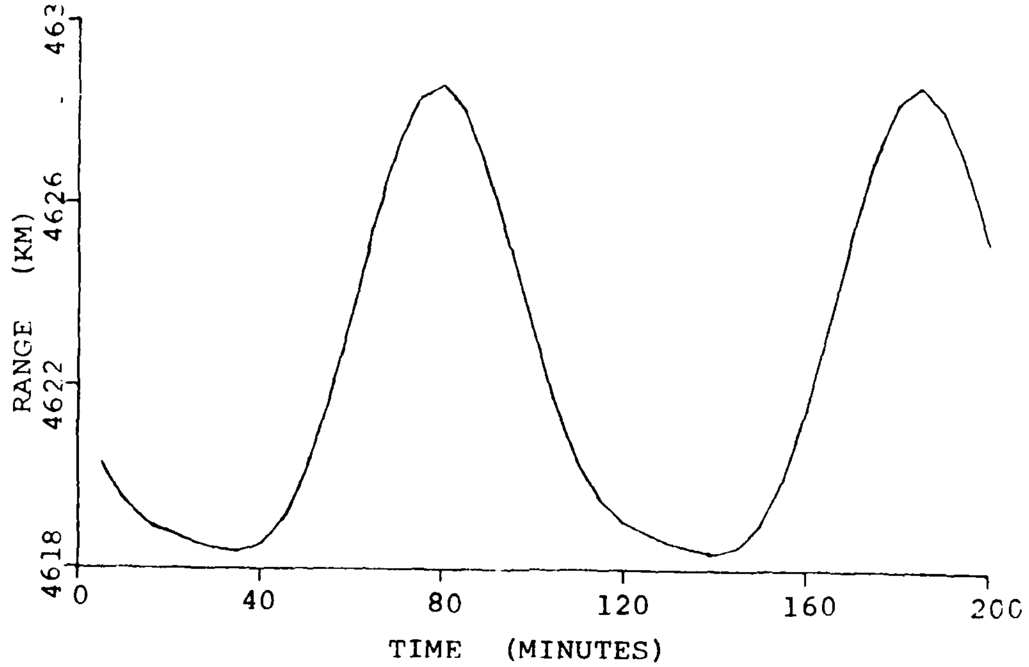

Fig. 8. Unperturbed Motion with Slightly Eccentric Orbits

change in range between two satellites orbiting within the same orbit is a result of the oscillation in the orbit's eccentricity caused by the Earth's oblateness perturbation.

Examination of Figure 7 also reveals a tendency for the range oscillations to be nonsymmetric about the average value rather than being purely sinusoidal. Equation (31) shows that the time rate of change in the orbit's eccentricity is a function of the satellite's true anomally, θ. However, true anomally is based upon the argument of perigee, which will also change with time when the orbit is perturbed as shown in the next equation (2:405).

$$\frac{d\omega}{dt} = - \frac{\sqrt{1-e^2}\cos\theta}{nae} F_x + \frac{\ell}{eh}\left[\sin\theta\left(1 + \frac{1}{1+e\cos\theta}\right)\right] F_y - \frac{r\cot i \sin(\omega+\theta)}{na^2\sqrt{1-e^2}} F_z \qquad (32)$$

As with Equation (31), this equation is unstable when eccentricity is zero, but serves to show that the argument of perigee is influenced by local force components and will change over time. The result of gravitational perturbations on the argument of perigee is a rotation of the line of apsides. The magnitude and direction of this rotation depends upon the orbit's radius and inclination (2:158). Because of the apsidal rotation, it was hypothesized that the location of the asymmetry in the plot of range versus time was dependent upon where the argument of perigee was defined in the initial conditions.

To test this hypothesis the baseline case was run several times using different initial values for the argument of perigee, but with the satellite always at the same starting point in the orbit. However, the results from all of these test cases yielded the same plot for range versus time, leading to the conclusion that the asymmetry in the range versus time plot was independent of the argument of perigee. Therefore, several additional test cases were formulated that used a common argument of perigee but started with the satellites at various locations throughout the orbit. Calculation of where the asymmetry and the maximum/minimum range values occurred relative to the ascending node showed no consistent location for any of these three conditions.

The only definite conclusions which can be drawn from this series of test cases are that the range between two satellites located in the same orbit is oscillatory, the oscillations repeat with a frequency equal to one orbital period, and the magnitude of the oscillations neither damps out nor diverges with time.

To analyze the sensitivity of the baseline results to variations in orbital parameters, several test cases were run using different combinations of orbital radius and inclination. The initial conditions for all of the first scenario orbits are summarized in Table I.

The results for all of these cases showed the same general trend as the baseline case discussed previously. Because the average value for the range between the satellites was not equal to the calculated range for a separation of 36 degrees, the plots of range versus time are difficult to compare directly. Therefore, to see the magnitude of the oscillations only, the results were normalized by subtracting each test case's minimum range from the calculated range at each time step during that run. These normalized results are shown in Figures 9 through 11, where squares indicate the 40° inclination cases, octagons

TABLE I

INITIAL ORBITAL ELEMENTS, FIRST SCENARIO

Case	ℓ	e	i	Ω	ω
1	7378 km	0	40°	45°	0°
2*	7378 km	0	60°	45°	0°
3	7378 km	0	80°	45°	0°
4	6678 km	0	40°	45°	0°
5	6678 km	0	60°	45°	0°
6	6678 km	0	80°	45°	0°
7	12378 km	0	40°	45°	0°
8	12378 km	0	60°	45°	0°
9	12378 km	0	80°	45°	0°

* Baseline orbit.

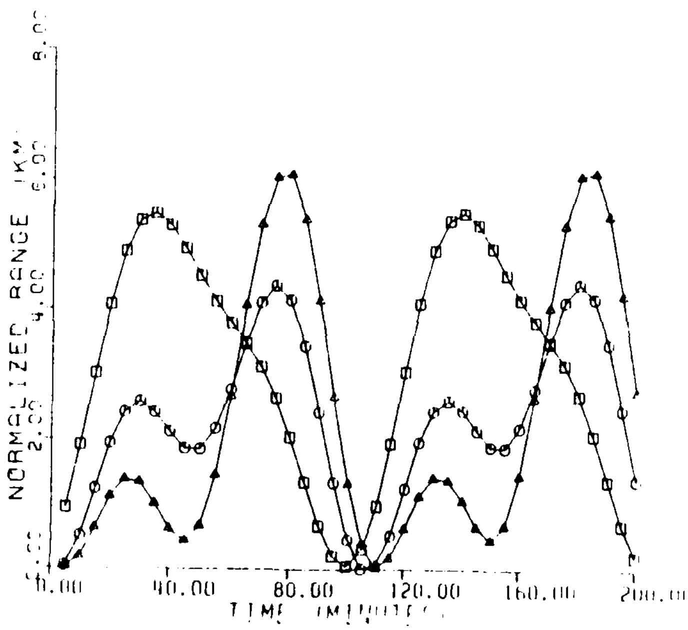

Fig. 9. Normalized Range vs. Time; Cases 1, 2, and 3

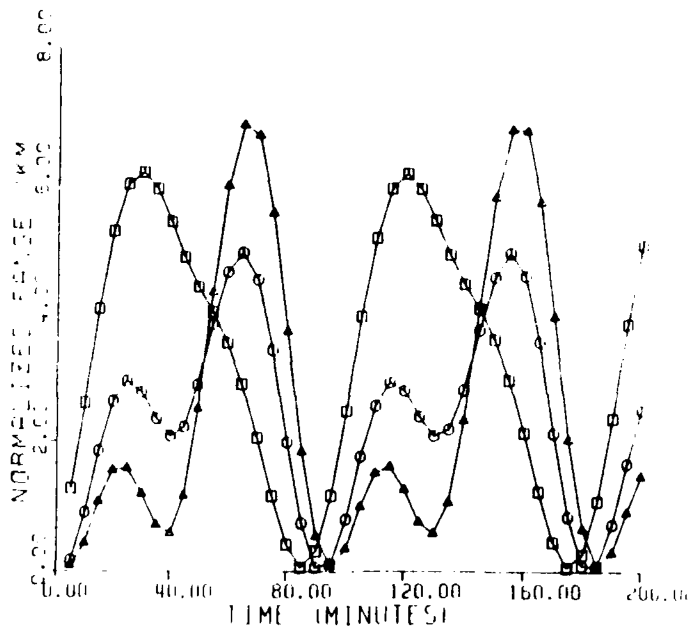

Fig. 10. Normalized Range vs. Time; Cases 4, 5, and 6

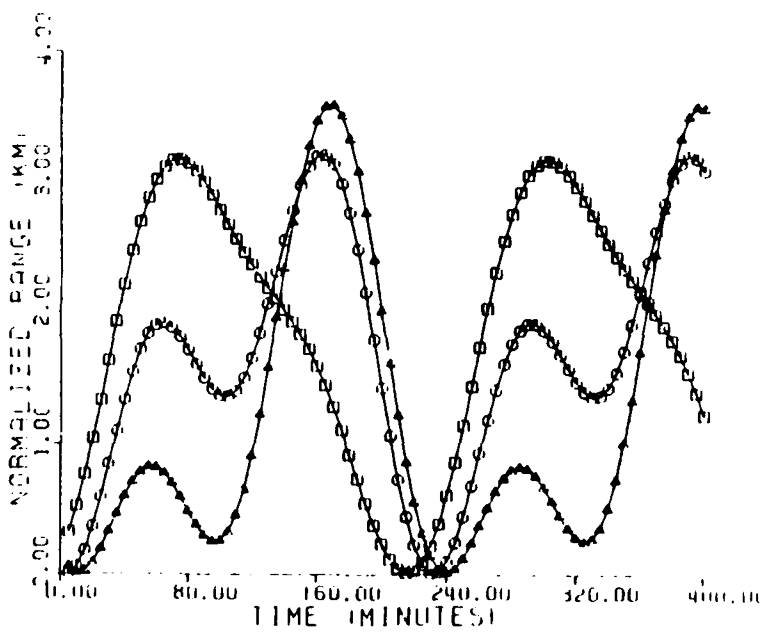

Fig. 11. Normalized Range vs. Time; Cases 7, 8, and 9

indicate 60° cases and triangles indicate 80° cases. The total magnitude of the oscillations ranged from 2.5 km to 6.1 km which, for all cases, is a deviation of less than two-tenths of 1 percent of the average value.

Second Scenario

The second scenario involved evaluating the relative motion between two satellites in similar, but different, orbits with ascending nodes spaced 36 degrees apart. Again, this spacing equates to the ten orbit constellation originally proposed as the baseline condition. Initial conditions for the nine test cases run for the second scenario are outlined in Table II, with Case 11 using the baseline orbit conditions.

TABLE II

INITIAL ORBITAL ELEMENTS, SECOND SCENARIO

Case	ℓ	e	i	ω	Ω_1	Ω_2
10	7378 km	0	40°	20°	10°	46°
11*	7378 km	0	60°	20°	10°	46°
12	7378 km	0	80°	20°	10°	46°
13	6678 km	0	40°	20°	10°	46°
14	6678 km	0	60°	20°	10°	46°
15	6678 km	0	80°	20°	10°	46°
16	12378 km	0	40°	20°	10°	46°
17	12378 km	0	60°	20°	10°	46°
18	12378 km	0	80°	20°	10°	46°

* Baseline conditions.

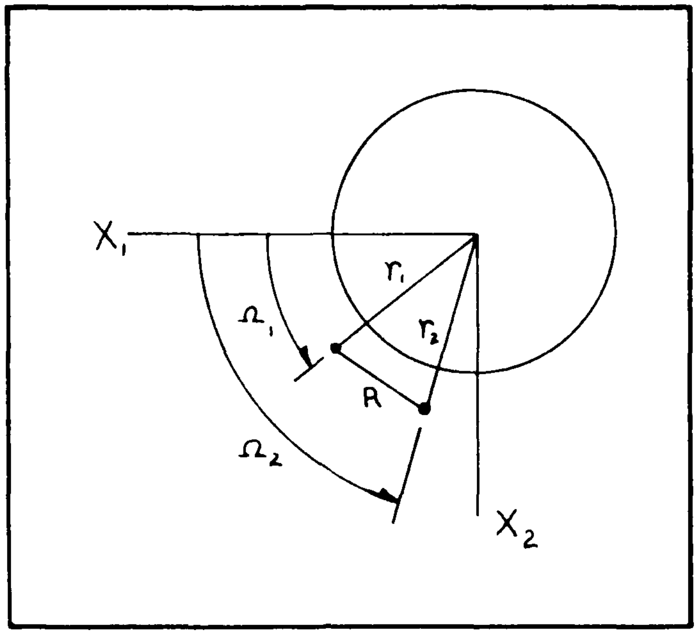

Fig. 12. Illustration of Second Scenario Initial Conditions

Unlike the in-plane arrangements evaluated under the first scenario, the range between satellites in different orbit planes will oscillate between some maximum and minimum value even for unperturbed motion. This can be illustrated by a simple example.

Suppose two satellites are initially positioned over the equator (at the ascending node) in circular orbits that are both at 90 degrees of inclination. If the nodes of the two orbit planes are separated by some amount, the initial range can be visualized as illustrated in Figure 12.

The range at this point can be calculated as

$$R = 2\, r \sin(\Delta\Omega/2)$$

$$= 2\ (7378)\ \sin(18) = 4559 \text{ km} \qquad (33)$$

As the satellites progress in their orbits from this initial position, they will arrive over the pole simultaneously after one-fourth of a period, at which time the range between them will be zero. From this point the range will again increase to a maximum as the satellites reach the equator at the descending node. This pattern will repeat every half period. For orbits at inclinations other than 90 degrees, the same general pattern will occur for range versus time but the minimum range value will be greater than zero and dependent upon the inclination.

For the baseline orbits inclined at 60 degrees with a radius of 7378 km, the minimum range was calculated by finding the inertial position vectors for the two satellites from the orbital elements after one quarter period. This is done by transforming each position vector expressed in its own perifocal reference frame into the inertial frame (2:82,83) and then finding the magnitude of the relative position vector. This calculation resulted in an unperturbed minimum range of 2279.972 km for the baseline case.

The results for the second scenario baseline case are shown in Figure 13. The maximum range for this case was 4559.543 km and the minimum was 2279.730 km.

Figure 14 is a plot of range versus time for the unperturbed, baseline case. The maximum range for this case was 2279.973 km and the minimum was 4559.932 km. Comparison with Figure 13 shows that both cases are stable over time, with a difference between maximum and minimum range of 2279.959 km for the unperturbed case and 2279.813 km with perturbations.

Because of the large excursions in range present even in the absence of perturbations, the results are somewhat difficult to interpret. Because the range rate for the second scenario cases was always on the order of hundreds or thousands of kilometers per second, it was not possible to synchronize the results of the perturbed and unperturbed cases to allow the deviations to be calculated. However, a comparison of the range versus time plots for the perturbed and unperturbed cases shows that the difference between these two cases is insignificant.

As was done with the first scenario cases, the baseline case for the second scenario was tested for sensitivity to changes in orbital altitude and inclination using the initial conditions shown in Table II.

The results of these additional eight cases are shown in Figures 15 through 17, and as with the baseline

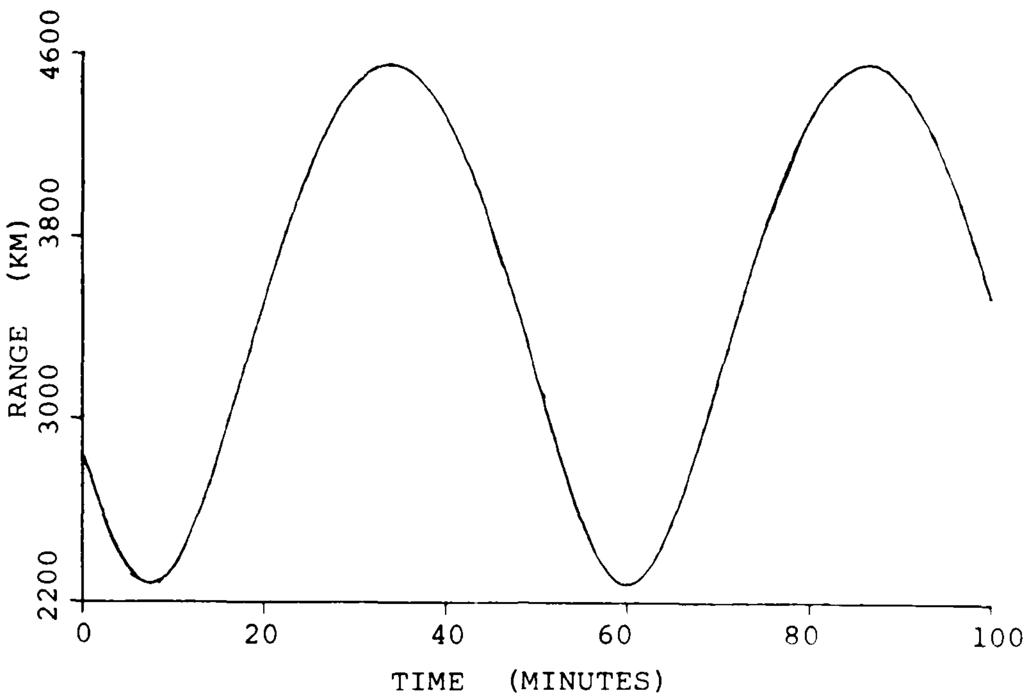

Fig. 13. Baseline Conditions, Second Scenario

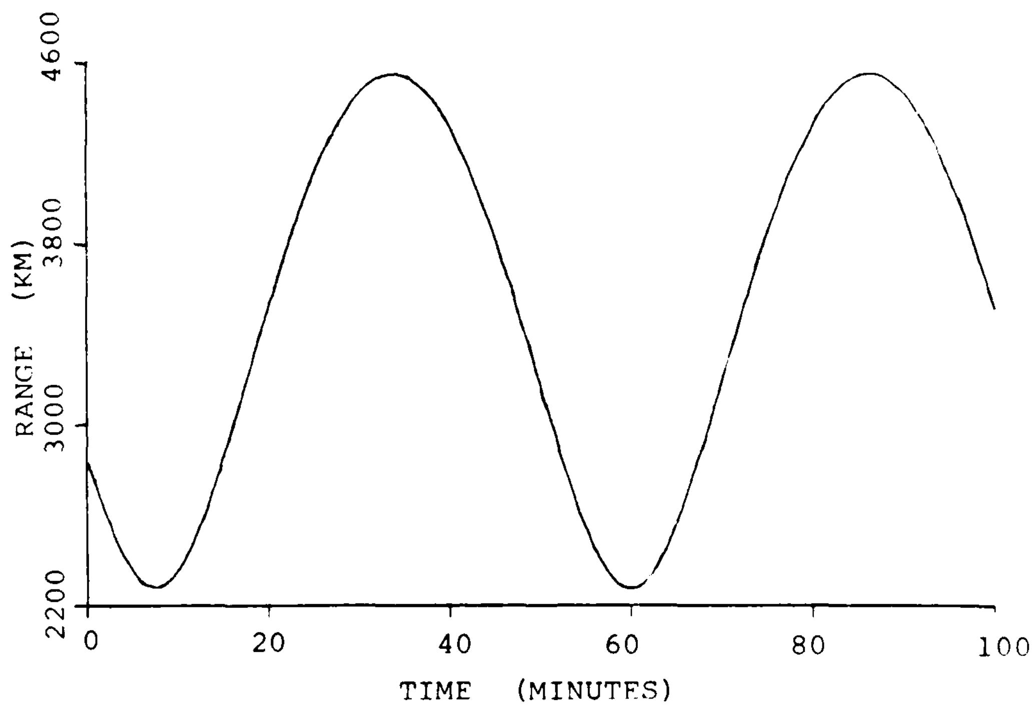

Fig. 14. Unperturbed Baseline Conditions, Second Scenario

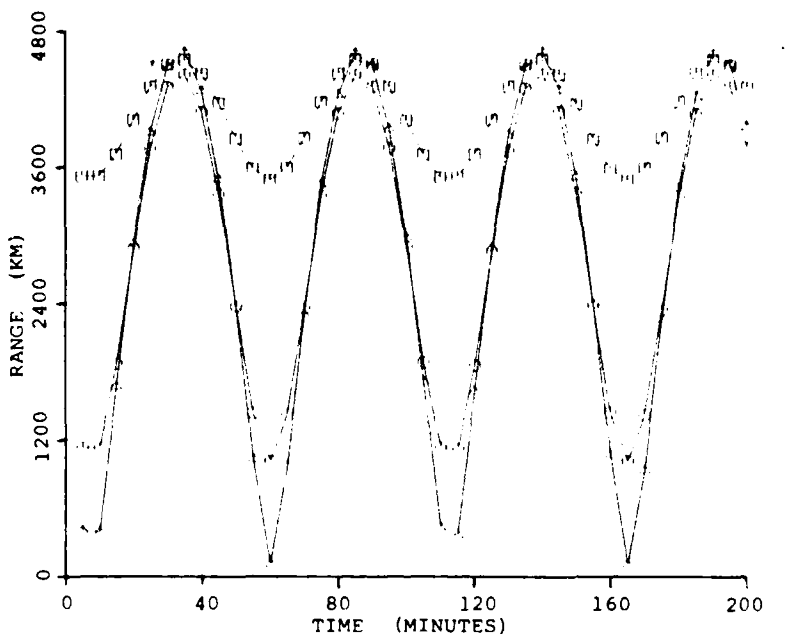

Fig. 15. Range vs. Time; Cases 10, 11, and 12

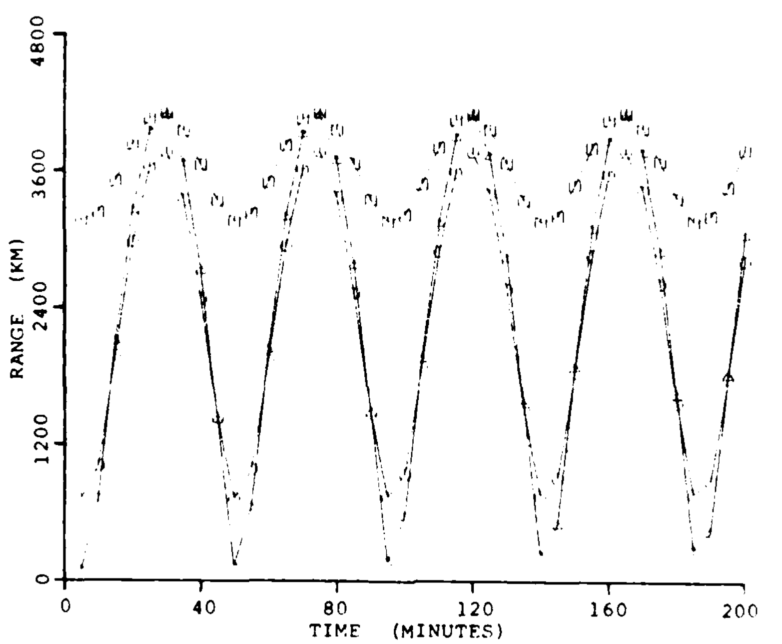

Fig. 16. Range vs. Time; Cases 13, 14, and 15

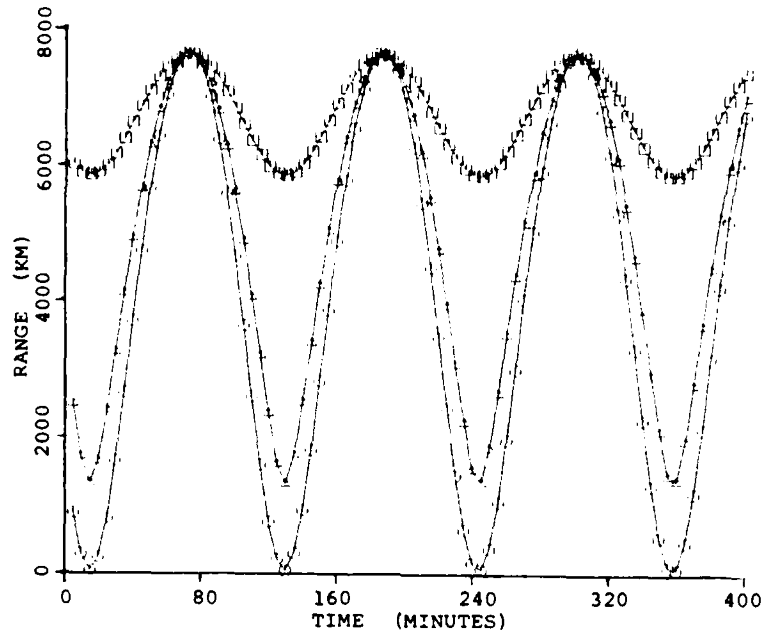

Fig. 17. Range vs. Time; Cases 16, 17, and 18

case the range oscillations show no tendency to change with time. Again, squares correspond to 40° inclination orbits, octagons indicate 60° orbits and triangles indicate 80° orbits. This indicates that the relative motion is stable, and, as calculated for the baseline condition, the deviations from two-body motion are minor.

IV. Conclusions and Recommendations

Conclusions

The basic question this research effort attempted to answer was whether the perturbations caused by the Earth's non-uniform gravitational field on the relative positions of satellites arranged as a constellation are large enough to significantly alter the constellation's operational capabilities. Relative motion was calculated using a set of equations for the variation of the difference between the orbital elements of two satellites. These equations allow the inclusion of any arbitrary perturbing forces, and such forces are introduced into the equations as components of the force vector expressed in a reference frame centered at the satellite. The perturbing force used for this study was the J_2 harmonic term of the Earth's geopotential function.

Two basic scenarios were evaluated for the study. The first scenario looked at the relative motion between two satellites orbiting in the same orbit plane, and the second scenario calculated relative motion between satellites in similar orbits but different orbital planes.

The results presented in the previous chapter show only small oscillations in range between adjacent satellites within one orbit or between satellites in adjacent orbits. The magnitude of the oscillations in all cases was essentially equal regardless of the initial orbit or the initial location of the satellite within the orbit, and never exceeded more than .2 percent of the average range value in any case investigated. This basic result showed only minor deviations due to changes in altitude or inclination.

Oscillations in range between two satellites which provide some type of coverage, or footprint, on the Earth's surface or upper atmosphere will result in an even smaller oscillation of the footprint at the Earth's surface as illustrated in Figure 18 where R_p is the range between the subpoints of the satellites, R_s is the range between the satellites, r_e is the radius of the Earth, and r_s is the length of the satellite's radius vector.

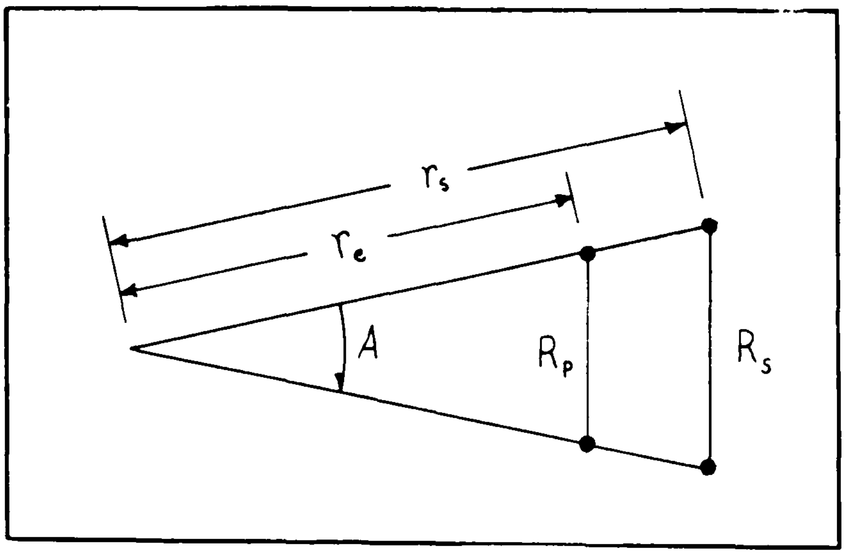

Fig. 18. Relationship Between Satellite Range and the Range of Their Subpoints

Because of similar triangles, R_p can be found from the ratio

$$\frac{R_p}{R_s} = \frac{2\ r_e \sin(A/2)}{2\ r_s \sin(A/2)} \tag{34}$$

so that

$$R_p = R_s \frac{r_e}{r_s} \tag{35}$$

Similarly, oscillations in range between satellites will result in oscillations in the range between the subpoints.

$$\Delta R_p = \Delta R_s \frac{r_e}{r_s} \tag{36}$$

Since r_s will always be greater than r_e, ΔR_p will always be less than ΔR_s. When combined with the fact that the range oscillations are in no case larger than 4 km, the effect of perturbation induced relative motion on a satellite constellation appears to be insignificant. The designer of a satellite constellation should be able to handle motion of adjacent weapons or sensor footprints by using only small amounts of overlap in the coverages.

Keeping in mind that the geopotential model used in this study was only latitude dependent, the findings of this investigation point to the conclusion that perturbations caused by the Earth's non-uniform gravitational field on the relative positions of satellites arranged as a constellation are insignificant and do not need to be considered as a constraint on the design of such constellations.

Recommendations for Further Study

While the J_2 harmonic used in the geopotential model has the largest magnitude of any of the harmonic coefficients, resonance caused by longitudinal variations in the earth's gravity field are possible, but no longitudinal dependence was modeled in this study. Therefore, expansion of the geopotential model to include major sectoral and/or tesseral harmonics provides an avenue for further investigation of this problem which may have significant results.

Appendix: Geopotential Expansion Theory

The dominant cause of gravitational perturbations on earth satellites is the bulge at the Earth's equator. This physical anomally shows up in the geopotential expansion of Equation (9) as the term for n=2, m=0, and can be written as

$$V = GMR_e^2 \, P_2^0(\cos \ ) \; C_{20}/r^3 \qquad \text{(A-1)}$$

since sin(m:) vanishes and cos(m:) equals one for m=0.

The terms $P_n^m(\cos \ )$ are known as the Legendre polynomials, and for this case can be solved so that

$$P_2^0(\cos \ ) = (3\cos^2 \ -1)/2 \qquad \text{(A-2)}$$

Substituting this into Equation (A-1) results in the potential being a function of latitude only, and terms of this type are known as zonal harmonics.

The harmonic coefficients C_{nm} and S_{nm} are derived empirically from satellite observations with coefficients of the form C_{n0} usually written as J_n. The reported value of J_2 is 1.0827×10^{-3} (9:290) with the values of all the other harmonic coefficients being on the order of 10^{-6} or less. Thus the J_2 harmonic has the largest effect on gravitational perturbations.

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A179 509

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1a. REPORT SECURITY CLASSIFICATION
UNCLASSIFIED

1b. RESTRICTIVE MARKINGS

2a. SECURITY CLASSIFICATION AUTHORITY

2b. DECLASSIFICATION / DOWNGRADING SCHEDULE

3. DISTRIBUTION / AVAILABILITY OF REPORT
Approved for public release;
distribution unlimited

4. PERFORMING ORGANIZATION REPORT NUMBER(S)
AFIT/GSO/AA/86D-1

5. MONITORING ORGANIZATION REPORT NUMBER(S)

6a. NAME OF PERFORMING ORGANIZATION
School of Engineering

6b. OFFICE SYMBOL *(If applicable)*
AFIT/ENY

7a. NAME OF MONITORING ORGANIZATION

6c. ADDRESS *(City, State, and ZIP Code)*
Air Force Institute of Technology
Wright-Patterson AFB, Ohio 45433-6583

7b. ADDRESS *(City, State, and ZIP Code)*

8a. NAME OF FUNDING / SPONSORING ORGANIZATION
Systems Concepts Group

8b. OFFICE SYMBOL *(If applicable)*
AFWAL/AART-2

9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER

8c. ADDRESS *(City, State, and ZIP Code)*
AF Wright Aeronautical Labs
Wright-Patterson AFB, OH 45433-6543

10. SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO	WORK UNIT ACCESSION NO.

11. TITLE *(Include Security Classification)*
An Analysis of Geopotential Effects on Satellite Constellations

12. PERSONAL AUTHOR(S)
Mark J. Buechter, Captain, USAF

13a. TYPE OF REPORT
MS Thesis

13b. TIME COVERED
FROM ________ TO ______

14. DATE OF REPORT *(Year, Month, Day)*
1985, December

15. PAGE COUNT

16. SUPPLEMENTARY NOTATION

17. COSATI CODES

FIELD	GROUP	SUB-GROUP
22	03	

18. SUBJECT TERMS *(Continue on reverse if necessary and identify by block number)*
Satellite Constellations; Satellite Networks; Satellite Constellation Design, Spacecraft Trajectories; Relative Motion of Spacecraft

19. ABSTRACT *(Continue on reverse if necessary and identify by block number)*

Thesis Advisor: Joseph W. Widhalm, Lt Col, USAF
Assistant Professor of Aerospace Engineering

Approved for public release: IAW AFR 190-17.
Development

20. DISTRIBUTION / AVAILABILITY OF ABSTRACT
☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS

21. ABSTRACT SECURITY CLASSIFICATION
UNCLASSIFIED

22a. NAME OF RESPONSIBLE INDIVIDUAL
Joseph W. Widhalm, Lt Col, USAF

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22c. OFFICE SYMBOL
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Current plans for systems to be used for ballistic missile defense sometimes call for using satellites that are placed so as to form a constellation that can continuously monitor specfied areas of the Earth's surface or direct weapons against attacking missiles or warheads. This study analyzes the relative motion between satellites within such a constellation under gravitational perturbations caused by the Earth's equatorial bulge (oblateness). Relative motion is calculated using a system of equations which describes the variation of relative orbital elements between two satellites. The cases studied simulate the position of two satellites that are located within a constellation containing ten orbits with ten satellites in each orbit. The orbits investigated were all circular with altitudes ranging from 300 km to 1000 km and inclinations ranging from 40 degrees to 80 degrees. Range between the satellites was oscillatory with deviations from the average range of up to 5 km. The results vary only slightly with changes in orbital inclination or altitude. These results show that the relative motion between satellites in a low altitude constellation caused by the Earth's oblateness does not significantly affect the initial geometry of the constellation.

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